

$$Q. (D^4 - 81)y = 0$$

$$A.E. \quad m^4 - 81 = 0$$

$$(m^2)^2 - 9^2 = 0 \quad (m^2 + 9)(m^2 - 9)$$

$$m = 3i, -3i, 3, -3$$

$$y_c = e^{0x} (C_1 \cos 3x + C_2 \sin 3x) + C_3 e^{3x} + C_4 e^{-3x}$$

Q.1. Let e^x, e^{2x}, e^{3x} are components of LDE. Find the D.E.

Sol:

$$m = 1, 2, 3$$

$$A.E. \quad (m-1)(m-2)(m-3) = 0$$

$$m^3 - 6m^2 + 11m - 6 = 0$$

$$(D^3 - 6D^2 + 11D - 6)y = 0$$

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$$

Ans.

Q.2. If e^x, xe^x, e^{-x} are components of LDE then find the D.E.

$$\text{Root of A.E. } m = 1, 1, -1$$

$$\text{P.S. } y_p = \frac{1}{F(D)} v(x)$$

$$I) \quad \frac{1}{D} v(x) = \int v(x) dx$$

$$II) \quad \frac{1}{D-a} v(x) = e^{ax} \int e^{-ax} v(x) dx$$

if $f(a) \neq 0$

$$III) \quad \frac{1}{F(D)} e^{ax} = \frac{1}{F(a)}$$

if $f(a) = 0$

$$\frac{1}{F(D)} e^{ax} = x \frac{1}{dF(D)} e^{ax}$$

$$\frac{1}{F(D)} e^{ax} = x \frac{1}{d F(D)} e^{ax}$$

iv $\frac{1}{F(D)} x^m \rightarrow$ Expand $F(D)$ by Binomial thm in ascending powers of D .

$$v \frac{1}{F(D)} e^{ax} \cdot v(x) = e^{ax} \frac{1}{F(D+a)} v(x)$$

$$vi \frac{1}{F(D^2)} \sin ax = \frac{1}{F(-a^2)} \sin ax \quad F(-a^2) \neq 0 \quad D^2 \rightarrow -a^2$$

$$\frac{1}{D^2 + a^2} \sin ax = \frac{-x}{2a} \cos ax \quad F(-a^2) = 0 \quad F(D^2) \rightarrow F(-a^2) = 0$$

$$vii \frac{1}{F(D^2)} \cos ax = \frac{1}{F(-a^2)} \cos ax \quad F(-a^2) \neq 0$$

$$\frac{1}{D^2 + a^2} \cos ax = \frac{x}{2a} \sin ax \quad F(D^2) \rightarrow F(-a^2) = 0$$

$$Q. 2. (D^4 + 2D^3 - 3D^2) y = 3e^{2x} + 4 \sin x$$

$$\text{Exercise } y_c = (C_1 + C_2 x) e^{ax} + C_3 e^x + C_4 e^{-3x}$$

$$y_p = \frac{1}{F(D)} v(x) = \frac{1}{D^4 + 2D^3 - 3D^2} [3e^{2x} + 4 \sin x]$$

$$p-1. \text{ for } 3e^{2x} = \frac{1}{D^4 + 2D^3 - 3D^2} 3e^{2x} = 3 \frac{1}{2^4 + 2 \cdot 2^3 - 3 \cdot 2^2} e^{2x} = \frac{9}{20} e^{2x}$$

$$p-1. \text{ for } 4 \sin x = \frac{1}{D^4 + 2D^3 - 3D^2} 4 \sin x = 4 \frac{1}{D^4 + 2D^3 - 3D^2} \sin x$$

$$= 4 \frac{1}{-1-2-3} \sin x = 4 \frac{1}{-6} \sin x$$

$$= 4 \frac{1}{D^2(D^2+2D-3)} \sin x = 4 \frac{1}{-1^2(-1^2+2D-3)} \sin x$$

$$= \frac{4}{-1(2D-4)} \sin x = -2 \frac{1}{D-2} \sin x = -2 \frac{D+2}{D^2-4} \sin x$$

$$= -2 \frac{(D+2)}{-1^2-4} \sin x = \frac{2}{5} (D+2) \sin x = \frac{2}{5} [D \sin x + 2 \sin x]$$

$$= \frac{2}{5} [\cos x + 2 \sin x]$$

$$y = C.F + P.I. = e^{0x} (C_1 + C_2 x) + C_3 e^x + C_4 e^{-3x} + \frac{3}{20} e^{2x}$$

$$+ \frac{2}{5} [\cos x + 2 \sin x] \quad \underline{\text{Ans.}}$$

$$Q. (D^2+1)y = \cos x + e^x \sin x + x e^{2x}$$

$$\text{H.W. } y_c = (C_1 \cos x + C_2 \sin x)$$

$$\text{P.I. for } \cos x = \frac{1}{D^2+1} \cos x = \frac{x}{2} \sin x$$

$$\text{P.I. for } e^x \sin x = \frac{1}{D^2+1} e^x \sin x = e^x \frac{1}{(D+1)^2+1} \sin x$$

$$= e^x \frac{1}{D^2+2D+2} \sin x = e^x \frac{1}{-1^2+2D+2} \sin x = e^x \frac{1}{2D+1} \sin x$$

$$= \frac{x}{e} \frac{2D-1}{4D^2-1} \sin x = e^x \frac{2D-1}{4(-1)^2-1} \sin x = -\frac{e^x}{5} (2D-1) \sin x$$

$$= -\frac{e^x}{5} [2D \sin x - \sin x]$$

$$= -\frac{e^x}{5} [2 \cos x - \sin x]$$

$$\text{P.I. for } x e^{2x} = \frac{1}{D^2+1} x e^{2x} = e^{2x} \frac{1}{(D+2)^2+1} x$$

$$\Rightarrow \frac{e^{2x}}{e} \frac{1}{D^2 + 4D + 5} x = \frac{e^{2x}}{5} \left[\frac{1}{1 + \frac{D^2 + 4D}{5}} \right] x$$

$$\Rightarrow \frac{e^{2x}}{5} \left[1 + \frac{D^2 + 4D}{5} \right]^{-1} x = \frac{e^{2x}}{5} \left[1 - \frac{4D + D^2}{5} \right] x$$

$$\Rightarrow \frac{e^{2x}}{5} \left[x - \frac{4}{5} \right]$$